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ON SEMIPRIME GAMMA NEAR- RINGS

MEHSIN JABEL ATTEYA[1], ATHEER GHAZI HUSSEIN[2] & DALAL IBRAHEEM RASEN[3]

[1,3]Department of Mathematics, AL-Mustansiriyah University, College of Education, Iraq

[2]Department of Mathematics, Waset University, College of Education, Iraq

ABSTRACT

The main purpose of this paper is to study and investigate some results concerning permuting tri-derivations on semi prime Γ- near-rings M, when M be a 3-torsion free semi prime Γ-ring with satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all x,y, z∈M, α,β∈Γ.If there exists a permuting tri-derivation D:M×M×M→M, where d is the trace of D

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1. INTRODUCTION

Every ring is a right near-ring (resp. a left near-ring). But in general the converse is not true. A right near-ring (resp. a left near-ring) not be a ring. In [2] Bell and Mason introduced the notion of derivations in near-rings. They obtained some basic properties of derivations in near-rings. Then Mustafa [11] investigated some commutativity conditions for a G-near-ring with derivations. Cho [5] studied some characterizations of G-near-rings and some regularity conditions. In classical ring theory, Posner [9], Herstein [6], Bergen [4], Bell and Daif [1] studied derivations in prime and semi prime rings and obtained some commutativity properties of prime rings with derivations. In near ring theory, Bell and Mason [2], and also Cho [5] worked on derivations in prime and semi prime near-rings. Gamma rings were first introduced by Nabusawa [12] and then Barnes [13] generalized the definition of Γ-rings. The ideal and other definitions of the concepts in Γ-rings we refer to [13].Eduard Domi[14]proved ,that if ideal I of Γ-near-ring M is maximal, then it is prime or M Γ M = I. Kalyan Kumar Dey and Akhil Chandra Paul [15]proved, let N be a semi prime Γ-near-ring and let U be a nonzero N-subset of N. If a be an element of N(U) such that UΓaΓa = {0} (or aΓaΓU = {0}), where N(U) is the normalizer of U, then a = 0. Yong Uk Cho[16]proved, Every left ideal of a P(1,2) Γ-near-ring is an ideal,where M is said to be a P(r;m) Γ-near-ring if there exist positive integers r, m such that xrΓM = MΓxm for all x M. Kalyan Kumar Dey[17]proved ,let M be a 2-torsion-free semi prime Γ-ring and D : M → M be an additive mapping which satisfies D(xαx)= D(x)αx for all x ∈M, α ∈ Γ. Then D is a left centralizer. Young Bae Jun , Kyung Hokim and Yong Uk Cho[18]proved , if d is a Γ-derivation on M, then d(xγy) = d(x)γy + xγd(y), for all x, y ∈ M and γ ∈ Γ, where M is Γ-near-ring. As a generalization of near-rings, Γ-near-rings were introduced by Satyanarayana [23]. Booth (together with Groenewald) have studied several aspects in Γ-near-rings (see [19, 20, 21, 22]). Ozturk, Sapanci, Soyturk and Kim [24] studied on symmetric bi-derivations on prime Γ-rings. Some fruitful results of prime Γ-rings were obtained by them. Ozturk [25] obtained some properties concerning to the mapping permuting tri-derivations on prime and semi prime Γ-rings. Permuting tri-derivations in prime and semiprime Γ-rings had been studied by Sapanci, M.A. Ozturk and Y.B. Jun [26]. Some remarkable results of these Γ-rings were obtained by them.

In this paper is to study and investigate some results concerning permuting tri-derivations on semiprime Γ- near-rings and prime Γ- near-rings, we give some results about that.

2. PRELIMINARIES

Throughout this paper, M will represent a Γ-near-ring is a triple (N, +, Γ) where

- (N, +) is a group (not necessarily abelian),
- Γ is a non-empty set of binary operations on N such that for each α Γ, (N, +, α) is a left near-ring.
- $a\alpha(b\beta c) = (a\alpha b)\beta c$, for all a, b, c N and α, β Γ

Γ–near-ring N is called a prime Γ-near-ring if N has the property that for a,b N, $a\Gamma N\Gamma b = \{0\}$ implies a = 0 or b = 0. N is called a semi prime Γ-near-ring if N has the property that for a N, a $\Gamma N\Gamma a = \{0\}$ implies a = 0. A nonempty subset U of N is called a right N-subset (resp. left N-subset) if UΓN U (resp. NΓU U), and if U is both a right N-subset and a left N-subset, it is said to be an N-subset of N. An ideal of N is a subset U of N such that (i) (U, +) is a normal subgroup of (N, +), (ii) aΓ (U + b) - aΓb U for all a ,b N, (iii) (U + a) Γb – a Γb U for all a, b N. If U satisfies (i) and (ii) then it is called a left ideal of N. If U satisfies (i) and (iii) then it is called a right ideal of N. On the other hand, a (two-sided) N-subgroup of N is a subset H of N such that (i) (H, +) is a subgroup of (N, +), (ii) NΓH H, and (iii) HΓN H. If H satisfies (i) and (ii) then it is called a left N-subgroup of N. If H satisfies (i) and (iii) then it is called a right N-subgroup of N. Note that normal N-subgroups of N are not equivalent to ideals of N. Every right ideal of N, right N-subgroup of N and right semi group ideal of N are right N-subsets of N, and symmetrically, we can apply for the left case. and Z(M) will be its center, the commutator $x\alpha y - y\alpha x$ will be denoted by $[x, y]\alpha$. We know that $[x\beta y, z]\alpha= x\beta[y, z]\alpha + [x, z]\alpha\beta y +x [\beta, \alpha]zy$ and $[x, y\beta z]\alpha = y\beta[x, z]\alpha + [x, y]\alpha\beta z + y[\beta,\alpha]xz$, for all x, y $z\in M$ and for all $\alpha, \beta\in\Gamma$. We shall using the assumption $x\alpha y\beta z=x\beta y\alpha z$ for all $x, y, z\in M$, $\alpha, \beta\in\Gamma$, the above identities reduce to $[x\beta y, z]\alpha= x\beta[y, z]\alpha + [x, z]\alpha\beta y$ and $[x, y\beta z]\alpha = y\beta[x, z]\alpha + [x, y]\alpha\beta z$, for all x, y $z\in M$ and for all $\alpha, \beta\in\Gamma$ which are used extensively in our results. Let U be a nonempty subset of M. Then a map d: M → M is said to be commuting (resp. centralizing) on U if $[d(x), x]\alpha = 0$ for all $x\in U$, and $\alpha\in\Gamma$ (resp. $[d(x), x]\alpha\in Z(M)$ for all $x\in U$, $\alpha\in\Gamma$), and is called central if $d(x)\in Z(M)$ for all $x\in M$ and $\alpha\in\Gamma$. Every central mapping is obviously commuting but not conversely in general, and d is called skew-centralizing on a subset U of M (resp. skew-commuting on a subset U of M) if $d(x)\alpha x + x\alpha d(x)\in Z(M)$ holds for all $x\in U$, $\alpha\in\Gamma$ (resp. $d(x)\alpha x + x\alpha d(x) = 0$ holds for all $x\in U$, $\alpha\in\Gamma$). Recall that M is said to be n-torsion free, where $n \neq 0$ is an integer, if whenever nx= 0, with $x \in M$ then x = 0. An additive map d: M→M is called a derivation $d(x\alpha y) = d(x)\alpha y + x\alpha d(y)$ for all $x, y\in M$, $\alpha\in\Gamma$. By a bi-derivation we mean a bi-additive map D: M × M → M (i.e., D is additive in both arguments), which satisfies the relations $D(x\alpha y, z) = D(x, z)\alpha y + x\alpha D(y, z)$ and $D(x, y\beta z) = D(x, y)\beta z + y\beta D(x, z)$ for all $x, y\in M$ $\alpha, \beta\in\Gamma$. Let D be symmetric, that is D(x, y) = D(y, x) for the $x, y\in M$.

The map d: M → M defined by d(x) = D(x, x) for all $x\in M$ is called the trace of D. A map D: M × M × M → M will be said to be permuting if the equation D(x, y, z)= D(x, z, y)= D(z, x, y)= D(y, z, x)= D (z, y, x) for all $x, y, z\in M$. A map d: M → M defined by d(x) =D(x, x, x) for all $x\in M$, where D: M × M × M → M is a permuting map is called the trace of D. It is obvious that, in case when D: M × M × M → M is a permuting map which is also tri-additive (i.e., additive in each argument), the trace d of D satisfies the relation d(x + y) = d(x) + d(y) + 3D(x, x, y) + 3D(x, y, y) for all $x, y\in M$. Since we have D(0, y, z) = D(0 + 0, y, z) = D(0, y, z) + D(0, y, z) for all $y, z\in M$, we obtain D (0, y, z) = 0 for all $y, z\in M$. Hence we get D(0,y,z)=D(x – x, y, z) = D(x, y, z) + D(–x, y, z) = 0 and so we see that D (–x, y, z) = –D(x, y, z) for all x, y, $z\in M$. This tells us that d is an odd function. A tri- additive map D: M × M × M → M will be called a tri-derivation if the relations $D(x\alpha w, y, z) = D(x, y, z)\alpha w + x\alpha D(w, y, z)$, $D(x, y\alpha w, z) = D(x, y, z)\alpha w + y\alpha D(x, w, z)$ and D

$(x, y, z\alpha w) = D(x, y, z)\alpha w + z\alpha D(x, y, w)$ are fulfilled for all $x, y, z, w\in M$ and $\alpha\in\Gamma$.If D is permuting, then the above three relations are equivalent to each other.

3. THE MAIN RESULTS

Theorem 3.1

Let M be a 3-torsion free semi prime Γ-near-ring satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y, z\in M$, $\alpha,\beta\in\Gamma$. If there exists a permuting tri-derivation $D: M\times M \times M \rightarrow M$ such that d is an automorphism central on M, where d is the trace of D, then M is commutative.

Proof: For all $x\in M$, we have the relation

$d(x) \in Z(M)$ for all $x\in M$, then

$[d(x),x]\beta = 0$ (1)

Substituting x by x + y, we obtain

$[d(x),y]\beta + [d(y),x]\beta + 3[D(x, x, y),x]\beta + 3[D(x, y, y),x]\beta + 3[D(x, x, y), y]\beta + 3[D(x, y, y), y]\beta = 0$ for all $x, y\in M$, $\beta\in\Gamma$. (2)

Putting –x instead of x in (2) and comparing (2) with the result, we arrive at

$[D(x,y, y),x]\beta + [D(x, x,y),y]\beta = 0$ (3)

Since d is odd, we set x = x + y in (3) and then use (1) and (3) to get

$[d(y), x]\beta + 3[D(x, y, y),y]\beta = 0$ (4)

Let us write $y\alpha x$ instead of x in (4), we obtain

$[d(y),y\alpha x]\beta + 3[D(y\alpha x, y, y), y]\beta = y\alpha[d(y),x]\beta + 3d(y)\alpha[x,y]\beta + 3y\alpha[D(x,y,y),y]\beta = y\alpha([d(y),x]\beta + 3[D(x,y,y),y]\beta) + 3d(y)\alpha[x,y]\beta = 0$.

Then

$d(y)\alpha[x,y]\beta=0$ .Since d is automorphism, we obtain

$y\alpha[x, y]\beta= 0$. Replacing x by $y\alpha x$, we get

$y\alpha x\gamma[x, y]\beta = 0$ (5)

Again left-multiplying by x implies that

$x\alpha y\gamma[x,y]\beta=0$ (6)

Subtracting (5) and (6) with using M is semiprime Γ- near-ring, we obtain the required result.

By same method in Theorem 2.1, it is easy to proof the following results.

Corollary 3.2

Let M be a 3-torsion free semiprime Γ- near-ring satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all $x, y, z\in M$, $\alpha, \beta\in\Gamma$, and let U be a non -zero ideal of M. If there exists a permuting tri-derivation $D:M \times M \times M \rightarrow M$ such that d is an automorphism commuting on U, where d is the trace of D, then U is a non-zero commutative ideal.

Corollary 3.3

Let M be a non-commutative 3-torsion free prime $\Gamma-$ near-ring satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all x, y, $z\in M$, α, $\beta\in\Gamma$. If there exists a permuting tri-derivation $D:M \times M \times M \rightarrow M$ such that d is an automorphism commuting on M, where d is the trace of D, then D=0(resp.d=0).

Proof: From the relation (6) in the proof of Theorem3.1, replacing x by D with using our hypothesis that M is non-commutative prime Γ-ring , we obtain the request result.

Theorem 3.4

Let M be a 3-torsion free semi prime Γ near--ring satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all x, y, $z\in M,\alpha,\beta\in\Gamma$.If there exists a permuting tri-derivation $D:M \times M \times M \rightarrow M$ such that d is an automorphism centralizing on M, where d is the trace of D, then D is centralizing on M.

Proof: For all x, $y\in M$, β, $\delta\in\Gamma$, assume that $[d(x),x]\beta\in Z(M)$ for all

$x\in M$ and $\beta\in\Gamma$. (7)

Replacing x by x + y and again using (7), we obtain

$[d(x),y]\beta + [d(y),x]\beta + 3[D(x,x,y),x]\beta + 3[D(x,y,y),x]\beta + 3[D(x,x,y),y]\beta + 3 [D(x,y,y),y]\beta\in Z(M)$ for all x, $y\in M$, $\beta\in\Gamma$. (8)

Replacing x by –x in (8) and compare (8) with the result to get

$[D(x,y,y),x]\beta + [D(x,x,y),y]\beta\in Z(M)$ for all x, $y\in M$, $\beta\in\Gamma$. (9)

Replacing x by x + y in (9) and using (9), we obtain

$[d(y),x]\beta + 3[D(x,y,y),y]\beta\in Z(M)$ for all x, $y\in M$, $\beta\in\Gamma$. (10)

Taking $x = y\alpha y$ in (10) and invoking (7), we get

$[d(y),y\alpha y]\beta + 3[D(y\alpha y,y,y),y]\beta = 8[d(y),y]\beta\alpha y\in Z(M)$ for all $y\in M$,

α, $\beta\in\Gamma$. (11)

Now commuting d(y) with (11), show that

$8[d(y),y]\beta\alpha[d(y),y]\beta = 0$ for all $y\in M$, α, $\beta\in\Gamma$.

Again substituting x by $y\alpha x$ in (10) gives

$[d(y),y\alpha x]\beta + 3[D(y\alpha x,y,y),y]\beta = y\alpha([d(y),x]\beta + 3[D(x,y,y), y]\beta) + 3d(y)\alpha [x,y]\beta+4[d(y),y]\beta\alpha x\in Z(M)$ for all x, $y\in M$, α, $\beta\in\Gamma$.

Then

$[y\alpha([d(y),x]\beta +3[D(x,y, y),y]\beta),y]\beta + [3d(y)\alpha[x,y]\beta + 4[d(y),x]\beta\alpha x,y]\beta = 0$ for all x, $y\in M$. And so we get

$3d(y)\alpha[[x, y]\beta, y]\beta + 7[d(y), y]\beta\alpha[x, y]\beta = 0$ for all x, $y\in M$. (12)

Since d acts as an automorphism with M is 3-torsion free the relation (12), reduces to $y\alpha[[x,y]\beta,y]\beta = 0$ for all x, $y\in M$, $\alpha,\beta\in\Gamma$

Replacing x by $r\delta x$, we get $y\alpha x\delta\, [[x,y]\beta,y]\beta + 2y\alpha[x,y]\beta = 0$ for all $x, y\in M$, $\alpha, \beta,\delta\in\Gamma$. (13)

Replacing y by $-y$ in (13) and subtracting with (13), gives

$4y\delta[x, y]\beta=0$ for all $x, y\in M$, $\beta, \delta\in\Gamma$. (14)

Replacing x by $x\gamma r$ and left-multiplying by s, we obtain

$4y\delta x\alpha[r, y]\beta = 0$ for all $x, y, r, s\in M$, $\alpha,\beta,\delta\in\Gamma$. (15)

Again in (2.14) replacing x by $x\lambda m$ and x by $s\delta x$, we get

$4y\gamma s\delta x\alpha[m, y]\beta=0$ for all $x, y, m, s\in M$, $\alpha, \beta, \delta, \gamma\in\Gamma$. (16)

Subtracting (15) and (16) with using M is 3-torsion free semiprime, we obtain $[s,y]\beta=0$ for all $s,y\in M$. Replacing s by d(s),we get

$[d(s),y]\beta=0$ for all $s,y\in M$. (17)

Substituting (17) in (10),gives

$3[D(x,y,y),y]\beta\in Z(M)$ for all $x, y\in M$, $\beta\in\Gamma$,then

$3[[D(x,y,y),y]\beta,s]=0$ for all $x, y,s\in M$, $\beta\in\Gamma$.Since M is 3-torsion free semi prime Γ near--ring, we obtain

$[D(x,y,y),y]\beta\in Z(M)$ for all $x, y\in M$, $\beta\in\Gamma$.

Thus, we obtain the required result.

Theorem 3.5

Let M be a 3-torsion free semiprime Γ− near- ring satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all $x, y, z\in M$, $\alpha, \beta\in\Gamma$. If there exists a permuting tri-derivation $D: M \times M \times M \to M$ such that d is commuting on M, where d is the trace of D, then

- d is a centralizing mapping on M
- d is a central mapping on M
- D is a central mapping on M

Proof: For proof (i) and (ii)From (17) in Theorem3.4 , we obtain the required result.(iii)By using same method in Theorem3.4 relation (17),we obtain $d(y)\in Z(M)$ for all $y\in M$, since d is the trace of D, then $d(y)=D(y,y, y)$ for all $y\in M$, which implies that $D(y,y,y)\in Z(M)$ for all $y\in M$. Thus, we completes the proof of the theorem.

Theorem 3.6

Let M be a non-commutative 3-torsion free prime Γ− near-ring. satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all $x, y, z\in M$, $\alpha, \beta\in\Gamma$. If there exists a permuting tri-derivation $D: M \times M \times M \to M$ such that d is commuting on M, where d is the trace of D, then D is commuting (resp. centralizing) on M.

Proof: Where, we say that d is commuting on M, then

We have the relation $[d(x), x]\beta = 0$ for all $x\in M$, $\beta\in\Gamma$.

The substitution of $x + y$ for x in above relation gives

$[d(x),y]\beta+[d(y),x]\beta+3[D(x, x, y), x]\beta+3[D(x, y, y), x]\beta+3[D(x, x, y),y]\beta + 3[D(x, y, y),y]\beta = 0$ for all $x, y\in M$, $\beta\in\Gamma$. (18)

Now, by the same method in Theorem 3.4, we arrive at

$y\delta[d(y), x]\beta + 3d(y)\delta[x, y]\beta + 3y\delta[D(x, y, y),y]\beta = 0$ for all $x, y\in M$, $\beta, \delta\in\Gamma$.

Which implies that $d(y)\delta[x, y]\beta = 0$ for all $x, y\in M$, $\beta,\delta\in\Gamma$.

Since M be a non-commutative prime Γ- near-ring, the above relation gives $d(y) \in Z(M)$ for all $x\in M$. By substitution the relation $d(y)\in Z(M)$ in (18) with using replacing x by y and M is 3-torsion free prime Γ- near-ring, we obtain $[D(y, y, y), y]\beta = 0$ for all $x, y\in M$, $\beta\in\Gamma$. Then D is commuting (resp. centralizing) of M.

Theorem 3.7

Let M be a 3-torsion free semiprime Γ- near-ring satisfying the condition $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M,\alpha,\beta\in\Gamma$. If there exists a permuting tri-derivation $D: M \times M \times M \rightarrow M$ such that d is skew-commuting on M, where d is the trace of D, then

- D is a central mapping on M.
- D is a central mapping on M.

Theorem 3.8

Let M be a 3-torsion free semi prime Γ- near-ring satisfying $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M,\alpha,\beta\in\Gamma$. If there exists a permuting tri-derivation $D:M\times M\times M \rightarrow M$ such that d is centralizing on M, where d is the trace of D, then d is commuting on M.

Proof: Assume that $[d(x), x]\beta\in Z(M)$ for all $x\in M$, $\beta\in\Gamma$ (19)

By linearization of (19), leads to

$[d(x),y]\beta + [d(y),x]\beta +3[D(x,x,y),x]\beta + 3[D(x,y,y),x]\beta + 3[D(x,x,y),y]\beta + 3[D(x,y,y),y]\beta\in Z(M)$, for all $x, y\in M$, $\beta\in\Gamma$. (20)

We substitute $-x$ for x in (20) and compare with (20), we get

$[D(x,y, y), x]\beta + [D(x,x,y),y]\beta\in Z(M)$, for all $x,y\in M,\beta\in\Gamma$. (21)

Putting $-x$ instead of x in (21) and comparing (21), with the result,

Replacing x by $x + y$ in (21) and using (21), we have

We get $[d(y),x]\beta +3[D(x,y,y),y]\beta\in Z(M)$, for all $x, y\in M$, $\beta\in\Gamma$. (22)

Taking $x = y\delta y$ in (22)and invoking (19), show that

$[d(y), y\delta y]\beta + 3[D(y\delta y, y, y), y]\beta = 8[d(y), y]\beta\delta y \in Z(M)$, for all $y\in M$, $\beta, \delta\in\Gamma$. (23)

Commuting the relation (23)with d(y) , gives

$8[d(y), y]\beta\delta[d(y),y]\beta = 0$, for all $y\in M$, $\beta, \delta\in\Gamma$. (24)

On the other hand, substituting x for $y\gamma x$ in (22), gives

$[d(y),y\gamma x]\beta + 3[D(y\gamma x, x, y),y]\beta = y\gamma[d(y), x]\beta + 3d(y)\gamma[x, y]\beta + 3[D(x, y, y), y]\beta + 4[d(y),y]\beta\gamma x\in Z(M)$ for all x, $y\in U$, $\beta, \gamma\in\Gamma$. (25)

Hence we have $[y\gamma\{[d(y), x]\beta + 3[D(x,y,y), y]\beta\gamma[x,y]\beta\}, y]\beta + [3d(y)\gamma[x, y]\beta + 4[d(y), y]\beta\gamma x, y]\beta = 0$ for all $x, y\in U$, $\beta, \gamma\in\Gamma$.

According to (22), we get $[3d(y)\gamma[x, y]\beta, y]\beta + 7[d(y), y]\beta\gamma[x, y]\beta = 0$, for all $x, y\in M$, $\beta, \gamma\in\Gamma$

Substituting $d(y)\lambda x$ for x in (23), it follows that

$d(y)\gamma\{3d(y)\lambda[[x, y]\beta, y]\beta + 7[d(y), y]\beta\gamma[x, y]\beta\} + 6d(y)\gamma[d(y),y]\beta\gamma[x, y]\beta + 7[d(y),y]\beta\gamma[d(y),y]\beta\gamma x=0$ for all x, $y\in M$, $\beta, \gamma\in\Gamma$, which by (25) implies

$6d(y)\gamma[d(y),y]\beta\gamma[x, y]\beta + 7[d(y),y]\beta\gamma[d(y),y]\beta\gamma x = 0$ for all $x, y\in M$, $\beta, \gamma\in\Gamma$. (26)

Letting $x = [d(y), y]\beta$ in (26) we arrive at $7[d(y),y]\beta\gamma[d(y),y]\beta\gamma[d(y),y]\beta = 0$ and so we get $7[d(y),y]\beta\gamma[d(y),y]\beta\gamma 7[d(y),y]\beta\gamma[d(y),y]\beta = 0$.

Since M is a semiprime Γ–near-ring, we obtain

$7[d(y),y]\beta\gamma[d(y),y]\beta = 0$ for all $x, y\in M$, $\beta, \gamma\in\Gamma$. (27)

Hence, the relations (23) and (27) yield $[d(y),y]\beta\gamma[d(y),y]\beta = 0$ for all $y\in M$, $\beta,\gamma\in\Gamma$. Since the center of a semiprime Γ- near-ring contains no nonzero nilpotent elements, we conclude that $[d(y),y]\beta = 0$ for all $y\in M$, $\beta\in\Gamma$.

In (21) replacing x by y with using above relation, we obtain

$3[[D(x,y,y),y]\beta ,s]=0$ for all $x, y,s\in M$, $\beta\in\Gamma$.

Since M is 3-torsion free semiprime Γ- near-ring, we obtain the required result.

REFERENCES

1. Bell H. E. and Daif M. N., On derivations and commutativity in prime rings, Acta. Math.Hungar. 66(4) (1995), 337-343.
2. Bell H. E. and Mason G., On derivations in near-rings. In: Gerhard Betsch (Ed.), Near-Rings and Near-Fields, Proceedings of the conference held at the University of Tubingen,Tubingen, August 4-10, 1985 (pp. 31-35). Noth-Holland, Amsterdam, 1987.
3. Bell H. E. and Mason G., On derivations in near-rings and rings. Math. J. Okayama Univ.34 (1992), 135-144.
4. Bergen J., Derivations in prime rings. Canad. Math. Bull. 26 (1983), no. 3, 267-227.
5. Cho, Y. U., A study on derivations in near-rings. Pusan Kyongnam Math. J., 12(1) (1996),63-69.
6. Herstein I. N., A note on derivations. Canad. Math. Bull. 21(3) (1978), 369-370.
7. Meldrum J. D. P., Near-Rings and Their Links with Groups, Research Notes inMathematics, 134. Pitman (Advanced Publishing Program), Boston-London-Melbourne, 1985. MR 88a:16068.
8. Pilz G., Near-rings, North-Holland Mathematics Studies, 23. North-Holland, Amsterdam, 1983.
9. Posner, E. C., Derivations in prime rings. Proc. Amer. Math. Soc. 8 (1957), 1093-1100.

10. Yong Uk Cho, Some conditions on derivations in prime near rings, J Korea Soc Math EducSer B Pure Appl Math 8(2) 2001, 145-152.
11. Mustafa Asci, Γ-(σ,β)-Derivation on Gamma Near Ring, International Math Forum, 2(3), 2007, 97-102.
12. N. Nabusawa, On a generalization of the Ring Theory, Osaka J. Math. 1 (1964), 65-75.
13. W. E. Barnes, On the G-rings of Nabusawa, Pacific J. Math. 18 (1966), 411-422.
14. Eduard Domi,Prime Ideals And Bi - Ideals In Gamma Near– Rings, 1st International Symposium on Computing in Informatics and Mathematics (ISCIM 2011) in Collaboration between EPOKA University and “Aleksander Moisiu” University of Durres on June 2-4 2011, 481-485.
15. Kalyan Kumar Dey and Akhil Chandra Paul, On Derivations in Prime Gamma-Near-Rings, GANIT J. Bangladesh Mathl. Soc. (ISSN 1606-3694) 32 (2012) 23-28
16. Yong Uk Cho, Some Results On Gamma Near-Rings, Journal Of The Chungcheong Mathematical Society, Vol.19, No.3, September 2006,225-229.
17. Kalyan Kumar Dey, Akhil Chandra Paul, and Isamiddin S. Rakhimov, Generalized Derivations in Semi prime Gamma Rings. International Journal of Mathematics and Mathematical Sciences. Hindawi Publishing Corporation. Volume 2012, Article ID 270132, 1-14, doi:10.1155/2012/270132
18. young Bae Jun , Kyung Hokim and Yong Uk Cho, On Gamma-Derivations in Gamma-Near-Ring, Soochow Journal Of Mathematics, Vol. 29, No. 3, pp. 275-282, July 2003.
19. G. L. Booth, Radicals in general Γ-near-rings, Quaestiones Mathematicae, 14 (1991), 117-127.
20. G. L. Booth, A note on Γ-near-rings, Studia Scientiarum Mathematicarum Hungarica, 23(1988), 471-475.
21. G. L. Booth and N. J. Groenewald, Equiprime Γ-near-rings, Quaestiones Mathematicae, 14 (1991), 411-417.
22. G. L. Booth and N. J. Groenewald, Matrix Γ-near-rings, Math. Japonica, 38:5(1993), 973-979.
23. Bh. Satyanarayana, Contributions to Near-ring Theory, Doctoral Thesis, Nagarjuna University, 1984.
24. M. A. Ozturk, M. Sapanci, M. Soyturk and K. H. Kim, Orthogonal traces on semi-prime gamma rings, Sci. Math. Jpn, 53, No. 3 (2001), 495-501.
25. Öztürk, M. A., "Permuting tri-derivations in prime rings", East Asian Math. J. Vol. 15 (2) (1999), 177-190.
26. M. Sapanci, M.A. Ozturk, Y.B. Jun, Symmetric bi-derivations on prime rings, East Asian Math. J., 15, No. 1 (1999), 105-109.